MA/MSCMT-06

June - Examination 2016

M.A./ M.SC. (Final) Mathematics Examination

Analysis and Advanced Calculus

Paper - MA/MSCMT-06

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Questions)

Note: Section 'A' contain 08 very short Answer type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Define convergence in normed linear space.

(ii) Explain weak convergence.

(iii) State open mapping theorem.

(iv) Define Hilbert space.

(v) Define orthogonal complement.

(vi) Define self-adjoint operators.

(vii) State spectral theorem for Hilbert space.

(viii) Write Taylor's formula with integral remainder.

Section - B $\qquad 4 \times 8 = 32$

(Short Answer Questions)

Note: Section 'B' contain 08 short answer type questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) It T be a linear transformation of a normed linear space N to normed linear space N′, then prove that inverse of T exists and is continuous on its domain of definition If and only If there exists a constant $K \geq 0$ S.T. $K \| x \| \leq \| T(x) \| \ \forall \ x \in N$

3) If M is a closed linear subspace of a normed linear space N and x_0 is a vector not in M then prove that there exist a functional F in conjugate space N^* such that $F(M) = \{0\}$ and $F(x_0) \neq 0$.

4) Prove that the mapping $\psi : H \to H^*$ defined by $\psi(y) = f_y$ where $f_y(x) = (x_1 y)$ for every $x \in H$ is an additive, one to one onto isometry but not linear.

5) If T is an operator on a Hilbert space H, then prove that $(Tx, x) = 0 \ \forall \ x \in H$ If and only If $T = 0$.

6) Let λ be an Eigen value of an operator T on a Hilbert space. If M_λ is the set consisting of all. Eigen vectors of T corresponding to Eigen value λ and the zero vector 0, then prove that M_λ is a non-zero closed linear subspace of H in variant under T.

7) If X and Y are any two Banach spaces over the same field K of scalers and V is an open subset of X. Let f : V$\rightarrow$ Y be continuous function and u_1 v be any two distinct points of V such that [u, v] $\subset$ V and f is differentiable in [u, v] then prove that $\| f(v) - f(\text{u}) \| \leq \| v - \text{u} \| \text{ Sup } \{\text{D}f(x) \| : x \in [\text{u}, v]\}$

8) If f be a regulated function on a compact interval (a, b) of R into a Banach space X over K (field of scalers) and g is continuous linear map of X into Banach space Y over K, then prove that gof is regulated and

$$\int_a^b gof = g\left(\int_a^b f\right)$$

9) State and prove Global uniqueness theorem for locally Lipschitz functions.

Section - C **2 × 16 = 32**

(Long Answer Questions)

Note: Section 'C' contain 04 long answer type questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words. Use of non-programmable scientific calculator is allowed in this paper.

10) Let P be the real number such that $| \leq p < \infty$, show that the space l_p^n of all n-Tuples of scalers with the norm defined by

$$\|x\|_p = \left\{\sum_{i=1}^{n} |x_i|^p\right\}^{1/p}$$ is a Banach space.

11) State and prove uniform boundness theorem.

12) If B is a Complex Banach space whose norm obeys parallelogram law and if an inner product is defined on B by
$u(x, y) = \|x + y\|^2 - \| x - y \|^2 + i \| x + iy\|^2 - i \| x - iy\|^2$ then prove that B is a Hilbert space.

13) If X and Y be Banach spaces over the same field K of scalers and V be an open subset of X. Let $f : V \to Y$ is twice differentiable at point $v \in V$, then prove that $D^2f(v) \in L(x^2,y)$ is a bilinear symmetric mapping that is for all $(x, y) \in XXY$
$D^2f(v).(x, y) = D^2f(v).(y, x)$